## DUAL DIG

## LEVEL II

2012

1. Find the missing entry in the following infinite sequence: 1001, 100, $\qquad$ $, 14,13,12,11,10,9,9,9, \ldots$
2. Find one-to-one functions $f$ and $g$ with the following characteristics (a and b are real numbers, $\mathrm{a}>0$ and $\mathrm{b}>0$ ):
a) $f(a+b)=f(a) \cdot f(b)$
b) $g(a \cdot b)=g(a)+g(b)$
3. Find the sum of the coefficients of all the terms after $(2 x+5 y)^{5}$ is expanded.
4. Calculate $\sum_{k=1}^{8}(-1)^{k+1} 3 k$
5. Suppose that $\ln 4=a$ and $\ln 9=b$. Write the following logarithm in terms of a and b : $\ln (36)^{7}$
6. Express $\sqrt{3}-i$ in polar form.
7. Find all the vertical asymptotes of the function: $f(x)=\frac{2 x^{2}+3 x}{x^{3}+2 x^{2}-5 x-6}$
8. Runners A, B, C, and D have chance of $0.3,0.2,0.1$, and 0.4 respectively of winning a race. If A drops out of the race, what is the probability that $B$ wins the race?
9. Solve: $x^{4}+5 x^{3}-27 x^{2}+31 x-10=0$
10. Determine all asymptotes of the function $\mathrm{f}(\mathrm{x})=\frac{x^{3}+2 x^{2}-15 x}{x^{2}-5 x-14}$
11. A survey is taken on methods of commuter travel. Each person checks bus, train, or car as a method of traveling to work. More than one pick is permitted. The results are:

| Bus | Train | Car |  <br> Train |  <br> Car | Train <br> \& Car | All <br> three |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 30 | 35 | 100 | 15 | 15 | 20 | 5 |

How many people completed the survey?
12. For a certain mammal, researchers have determined that the mesiodistal crown length of deciduous mandibular first molars is related to the post conception age of the tooth as $L(t)=-.015 t^{2}+1.4 t-7.5$, where $L(t)$ is the crown length (in millimeters) of the molar $t$ weeks after conception. Find the maximum length in mesiodistal crown of mandibular first molars during weeks 30 through 60 (round to three decimal places).
13. A power line is to be constructed from a power station at point A to an island at point C , which is 1 mile directly out in the water from a point B on the shore. Point $B$ is 4 miles down shore from the power station at A. It costs $\$ 5000$ per mile to lay the power line under water and $\$ 3000$ per mile to lay the line underground. At what point $S$ down shore from A should the line come to the shore in order to minimize cost? Note that $S$ could very well be B or A.
14. If $f(x)=\log \left(\frac{1+x}{1-x}\right)$ for $-1<\mathrm{x}<1$, then define $f\left(\frac{3 x+x^{3}}{1+3 x^{2}}\right)$ in terms of $f(x)$ :
15. In 2006, a team of archaeologists uncovered an undiscovered tomb. The archaeologists believe that the mummies are from the $18^{\text {th }}$ Dynasty, about 3300 to 3500 years ago. Determine the amount of carbon-14 that the mummies have lost. (note: the radioactive element carbon-14 has a half-life of 5750 years).
16. The area of a circle inscribed in a regular hexagon is $50 \pi$. Determine the area of the hexagon.
17. Decompose the following fraction into partial fractions: $\frac{4 x-13}{2 x^{2}+x-6}$
18. In the figure, it is given that angle $C=90^{\circ}, \overline{A D} \cong \overline{D B}, \overline{D E} \perp \overline{A B}, A B=20$, and $A C=12$. The area of quadrilateral ADEC is:

19. The distance from home plate to dead center field in a certain baseball stadium is 406 feet. A baseball diamond is a square with a distance from home plate to first base of 90 feet. How far is it from first base to dead centerfield? Round your answer off to the nearest foot.
20. Given 12 points in a plane no three of which are collinear, the number of lines they determine is:

